## Congruent Triangles

After today's lesson we should be able to use the properties of congruent triangles and prove triangles congruent by using the definition of congruence.

Vocabulary

1. Corresponding angles   angles that are in the same position in polygons with an equal number of sides.

2. Corresponding sides   sides that are in the same position in polygons with an equal number of sides.

(Size)   (Shape)

3. Congruent polygons: two polygons whose corresponding sides and   angles are congruent.

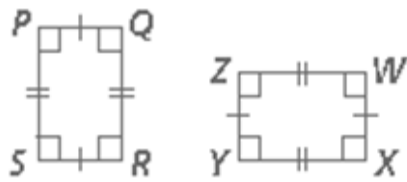$\triangle ABC \cong \triangle DEF$ → **Congruence Statement**
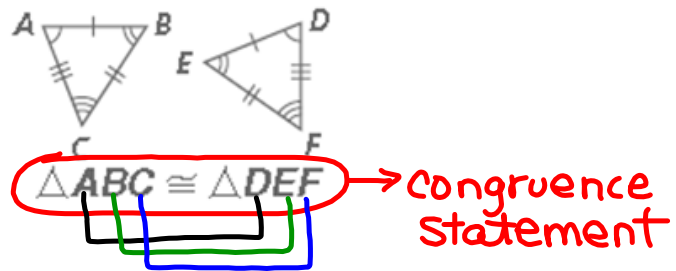
Name the pairs of corresponding sides:

$\overline{AB} \cong \overline{DE}, \ \overline{BC} \cong \overline{EF}, \ \overline{CA} \cong \overline{FD}$

Name the pairs of corresponding angles:

$\angle A \cong \angle D, \ \angle B \cong \angle E, \ \angle C \cong \angle F$

polygon *PQRS* ≅ polygon *WXYZ* ← congruence statement

Name the pairs of corresponding sides:

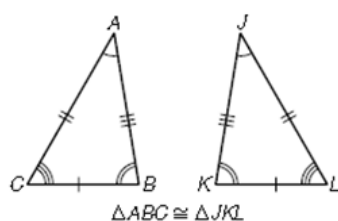$\overline{PQ} \cong \overline{WX}$, $\overline{RS} \cong \overline{YZ}$, $\overline{QR} \cong \overline{XY}$, $\overline{SP} \cong \overline{ZW}$

Name the pairs of corresponding angles:

$\angle P \cong \angle W$, $\angle Q \cong \angle X$, $\angle R \cong \angle Y$, $\angle S \cong \angle Z$
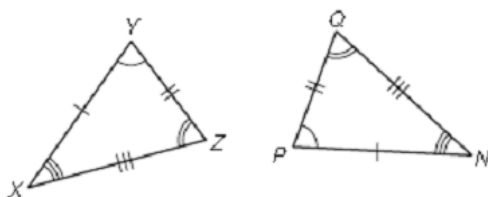
Triangles are **congruent** if they have the same size and shape. Their **corresponding parts**, the angles and sides that are in the same positions, are congruent.

$\triangle ABC \cong \triangle JKL$

Corresponding Parts	
Congruent Angles	Congruent Sides
$\angle A \cong \angle J$	$\overline{AB} \cong \overline{JK}$
$\angle B \cong \angle K$	$\overline{BC} \cong \overline{KL}$
$\angle C \cong \angle L$	$\overline{CA} \cong \overline{LJ}$

To identify corresponding parts of congruent triangles, look at the order of the vertices in the congruence statement such as $\triangle ABC \cong \triangle JKL$.

Given: $\triangle XYZ \cong \triangle NPQ$. Identify the congruent corresponding parts.

1. $\angle Z \cong$ $\angle Q$
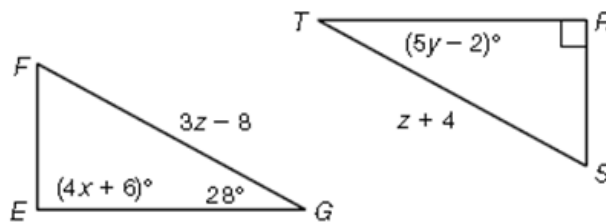
2. $\overline{YZ} \cong$ $\overline{PQ}$

3. $\angle P \cong$ $\angle Y$

4. $\angle X \cong$ $\angle N$

5. $\overline{NQ} \cong$ $\overline{XZ}$

6. $\overline{PN} \cong$ $\overline{YX}$

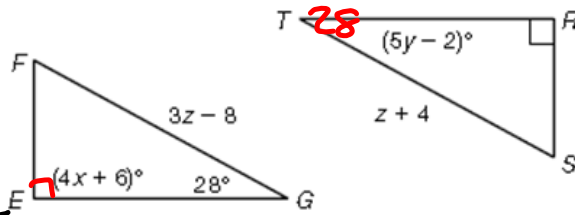Given: $\triangle EFG \cong \triangle RST$. Find each value below.

7. $x =$ _____21_____

$$4x + 6 = 90$$
$$\underline{\quad -6 \mid -6 \quad}$$
$$\frac{4x}{4} = \frac{84}{4}$$
$$X = 21$$

8. $y =$ _____6_____

$$5y - 2 = 28$$
$$\underline{\quad +2 \mid +2 \quad}$$
$$\frac{5y}{5} = \frac{30}{5}$$
$$y = 6$$

Given: $\triangle EFG \cong \triangle RST$. Find each value below.

$m\angle F + m\angle G = 90$
$m\angle F + 28 = 90$
$\underline{\quad -28 \quad -28}$
$m\angle F = 62$

9. $m\angle F = \underline{\quad\quad 62 \quad\quad}$

$m\angle F + m\angle E + m\angle G = 180$
$m\angle F + 90 + 28 = 180$
$m\angle F + 118 = 180$
$\underline{\quad\quad -118 \quad -118}$
$m\angle F = 62$

10. $ST = \underline{\quad\quad 10 \quad\quad}$

$ST = FG$
$z + 4 = 3z - 8$
$\underline{-z + 8 \quad -z + 8}$
$\dfrac{12}{2} = \dfrac{2z}{2}$

$6 = z$
$ST = 6 + 4 = 10$

(diagram: triangle with F at top, E bottom-left, G bottom-right; side FG = $3z - 8$, angle E = $(4x+6)°$, angle G = $28°$. Second triangle with T top-left marked 28, R top-right (right angle), S bottom-right; angle T = $(5y-2)°$, side TS = $z + 4$)

DEFINITION

TRIANGLE CONGRUENCE: Two triangles are congruent when all THREE pairs of corresponding sides are congruent (≅) AND all THREE pairs of corresponding angles are congruent (≅).
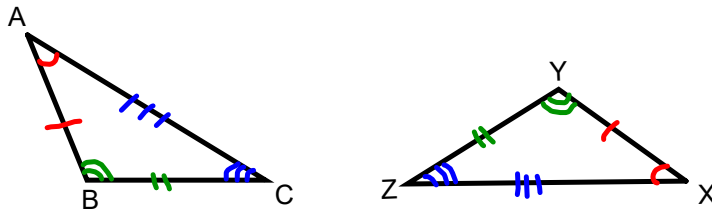
$$\left(\text{Sides} \quad \begin{array}{l} \overline{AB} \cong \overline{XY} \\ \overline{BC} \cong \overline{YZ} \\ \overline{CA} \cong \overline{ZX} \end{array} \quad + \quad \begin{array}{l} \angle A \cong \angle X \\ \angle B \cong \angle Y \\ \angle C \cong \angle Z \end{array} \quad \text{angles} \right)$$

$$= \triangle ABC \cong \triangle XYZ$$

You can prove triangles congruent by using the definition of congruence.

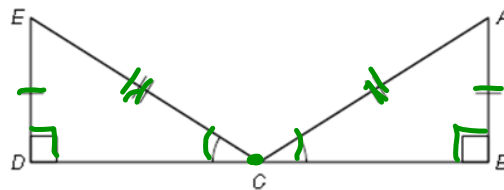**Given:** $\angle D$ and $\angle B$ are right angles.

$\angle DCE \cong \angle BCA$

C is the midpoint of $\overline{DB}$.

$\overline{ED} \cong \overline{AB}$, $\overline{EC} \cong \overline{AC}$

Prove: $\triangle EDC \cong \triangle ABC$

$\angle D \cong \angle B \qquad \angle E \cong \angle A$

$\overline{DC} \cong \overline{BC}$

(all right $\angle$s are $\cong$)

Proof:

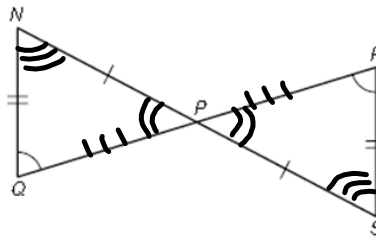| Statements	Reasons
1. $\angle D$ and $\angle B$ are rt. $\angle$s.	1. Given
2. $\angle D \cong \angle B$	2. Rt. $\angle \cong$ Thm.
3. $\angle DCE \cong \angle BCA$	3. Given
4. $\angle E \cong \angle A$	4. Third $\angle$ Thm.
5. C is the midpoint of $\overline{DB}$.	5. Given
6. $\overline{DC} \cong \overline{BC}$	6. Def. of mdpt.
7. $\overline{ED} \cong \overline{AB}$, $\overline{EC} \cong \overline{AC}$	7. Given
8. $\triangle EDC \cong \triangle ABC$	8. Def. of $\cong$ $\triangle$s

11. Complete the proof.

Given: $\angle Q \cong \angle R$

P is the midpoint of $\overline{QR}$.

$\overline{NQ} \cong \overline{SR}$, $\overline{NP} \cong \overline{SP}$

Prove: $\triangle NPQ \cong \triangle SPR$

Proof:

$\overline{QP} \cong \overline{RP}$

Statements	Reasons
1. $\angle Q \cong \angle R$	1. Given
2. $\angle NPQ \cong \angle SPR$	2. a. Vertical Angles Thm
3. $\angle N \cong \angle S$	3. b. Third Angles Thm
4. P is the midpoint of $\overline{QR}$.	4. c. Given
5. d. $\overline{QP} \cong \overline{PR}$	5. Def. of mdpt.
6. $\overline{NQ} \cong \overline{SR}$, $\overline{NP} \cong \overline{SP}$	6. e. Given
7. $\triangle NPQ \cong \triangle SPR$	7. f. Def. of $\cong$ $\triangle$s